Social Networks and the Effect of Reputation on Cooperation

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Abstract

In this paper we explore the management of trust relations, i.e., mechanisms that induce actors to place and to honor trust. These mechanisms can deter a trustee from abusing trust placed by the trustor, and thereby induce the trustor to place trust. We distinguish two types of embeddedness, namely, (1) repeated interactions between the same actors and (2) social networks that operate as information channels and link transactions of trustor and trustee to the trustee's relations with other trustors. In contrast with earlier game-theoretic models, networks are not assumed to be homogeneous. We focus on the relationship between network characteristics and the extent to which trust is placed.

1 Introduction

The governance of exchange relations is an important topic in different disciplines, such as economics, law, social psychology, and sociology. It is often not efficient to arrange complex transactions with extensive explicit contractual agreements. Durkheim (1893, Book I, Chapter 7) highlighted the limitations of such contracts. He addresses the importance of legal and extra-legal institutions to overcome unforeseen and unforeseeable future contingencies in an efficient manner. Qualitative empirical evidence that non-contractual modes of arranging transactions are indeed used extensively is presented in Macaulay (1963). Reputation is an important non-contractual mechanism for the governance of transaction. Recently, many contributions on this subject have been made by social network theorists (e.g., Granovetter, 1985) and by theoretical economists (e.g., Kreps and Wilson, 1982; Wilson, 1985; Kreps, 1990b). The notion behind reputation is that actors receive information about the behavior of their partner from third parties and use that information to decide how they are going to behave themselves.

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An example is a buyer-seller transaction. The seller has incentives to deliver a low quality product. The buyer has the option to demand a complete contract that eliminates the seller's incentives to deliver a low quality product. Such a contract, however, is costly both to the buyer and the seller. Thus, the 'no contracts' option is better for both the buyer and the seller. In this example, the buyer has no reason to believe that the seller will sell a high quality product if the transaction takes place without any relation to, e.g., future transactions or other conditions like commitments of the seller. Such an incentive structure leads to the inefficient outcome in which costly complete contracts are made. The concern about one's reputation could be one of the mechanisms that induces the seller to deliver a high quality product without the need for costly contracts. The reason is that if he sells a low quality product and receives an extra short term profit, the buyer informs other potential buyers. Consequently, these buyers may take precautionary means, such as, complete contracts, which implies a long term loss for the seller.

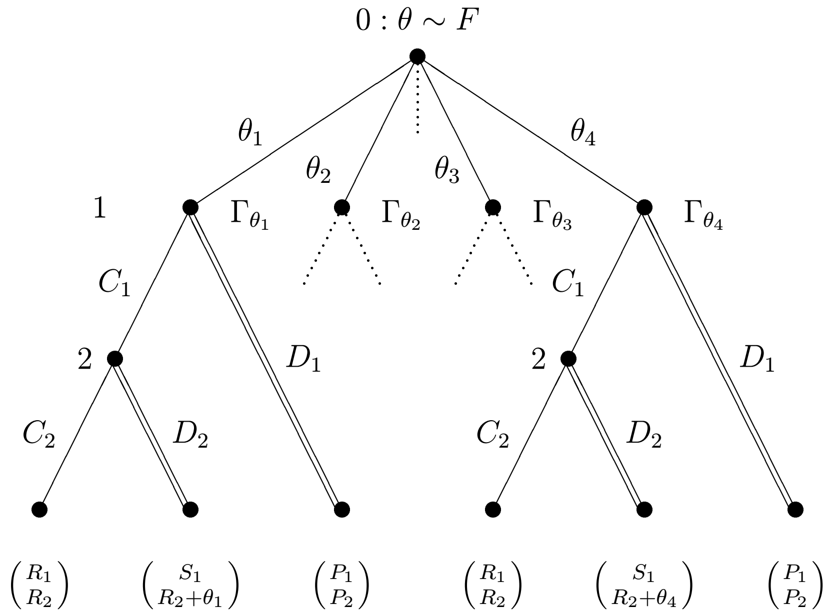
One essential point in modeling reputation is the transfer of information between actors. The most extreme cases, i.e., 'no transfer possible' or 'everybody knows what everybody is doing', are not satisfying. People receive information from others with whom they have some kind of relation. Therefore, we use social networks. The ties in the networks represent the relations in which information transfer is possible. The relevance of network embeddedness to reputation effects has been shown with game theoretical models in different studies (Raub and Weesie, 1990; Weesie et al. 1995). These models confirm findings from Coleman (1990, chs. 5 and 8) and Granovetter (1985), that the effect of reputation increases when the density of the network increases. These models assume, however, that the network between the actors is *homogeneous*. We assume that not only the density of a network is relevant, but also the specific structure of a network. Hence, we do not restrict our model to homogeneous networks. Rather, we study heterogeneous networks and, consequently, compare different structures in networks with the same density. At the end of the paper we address a problem of 'social engineering', i.e., we indicate how a network can be constructed with a fixed number of ties that is particularly efficient in terms of our model. The effect of reputation via network embeddedness on cooperative behavior in trust relations is studied in combination with some other relevant variables like payoffs, temporal embeddedness, i.e., future transactions with the same partner, and the use of contracts.

The remainder of this paper is organized as follows. Section 2 gives a detailed description of the model. In Section 3, some of the mathematical properties of the model will be analyzed and equilibria are identified. Section 4 examines how the equilibria depend on the model parameters. Properties of certain network structures are analyzed in Section 5. Finally, in Section 6 the main results are summarized and possibilities for further research are discussed. In an Appendix, the proves of the theorems are given together with some technical details.

2 The Model

The model developed in this paper is an Iterated Heterogeneous Trust Game (IHTG) $\Gamma(\Gamma_\theta, F, \boldsymbol{\delta}, \boldsymbol{\pi}, \boldsymbol{A}, w)$. First, we discuss the different elements in turn. The constituent game is a Heterogeneous Trust Game (HTG) Γ_θ. The HTG is a variant of the Trust Game for modeling simple trust relations (Dasgupta, 1988; Kreps, 1990b). The HTG

Figure 1 *Extensive form of the Heterogeneous Trust Game* Γ_θ, *where* $R_i > P_i, (i = 1, 2)$, $P_1 > S_1$, $\theta > 0$ *and* $\theta \sim F$.



is illustrated in Figure 1 in the extensive form. In the first stage of Γ_θ, a $\theta > 0$ is randomly generated from a distribution F by player 0 (nature). θ is the incentive for the trustee to abuse trust placed by the trustor. For technical reasons we assume that F is a continuous probability distribution with full support on $[0, \infty)$. An example of such a distribution with favorable analytical properties is $F_a(\theta) = Pr(\theta_t \leq \theta) = \theta/(a + \theta)$ (cf. Raub and Weesie, 1993; Weesie et al., 1995), where a is the median of F_a and so can be interpreted as the average incentive for opportunistic behavior. The arguments for studying a heterogeneous model are discussed below. The interacting trustor and trustee are informed on θ. In the second stage, the trustor chooses whether he wants to arrange the transaction with complete contracts (D_1) and both players receive a payoff P_i or if he trusts the trustee and arrange the transaction with incomplete contracts (C_1). In the third stage, the trustee has only a choice in case the trustor has placed trust. He can honor trust (C_2) and both players receive $R_i > P_i$, or abuse trust (D_2) in which case he will receive $R_2 + \theta > R_2$ and the trustor $S_1 < P_1$. Since $\theta > 0$, the only equilibrium of the one-shot game Γ_θ is $D_1 D_2$, which is inefficient because both actors prefer the $C_1 C_2$ outcomes.

The IHTG, $\Gamma(\Gamma_\theta, F, \boldsymbol{\delta}, \boldsymbol{\pi}, \boldsymbol{A}, w)$, is played by one trustee and k types of trustors. The game is played at discrete moments in time, $t = 0, 1, 2, \ldots$, and the trustee has at every moment a transaction with one trustor, during a random period. In accordance with the language of reputation models, the trustee is long-lived, while the trustor has at every moment a probability to 'die'. Only after one trustor dies a new trustor is introduced. During the alternation of trustors information can be exchanged according to the network between the different types of trustors.

A type can be interpreted as a group of trustors that is a segment of a certain market. For technical reasons we assume these segments to be infinitely large. The vector $\boldsymbol{\pi}$

gives the relative importance of the different segments. The proportion of trustors of type i equals π_i and $\sum_{j=1}^{k} \pi_j = 1$. These probabilities can have different interpretations. First, all trustors in different segments have the same probability of interactions with the trustee, but some segments contain more trustors than others. Second, the segments have the same size, however, trustors in some segments have more transactions with the trustee than trustors in other segments. Third, a combination of the two interpretations is possible.[1]

A trustor of type i survives with probability $1 - \delta_i$. (If $\delta_i = 1$, the trustor is called short-lived; if $\delta_i = 0$, he is called long-lived.). We assume that survival is independent of the history of the game. In particular, we do not model a relationship between a trustor and his (cumulative) payoff with the trustee. With probability δ_i, a trustor of type i dies. A new trustor of type j is chosen with probability $\pi_j > 0$ (possibly, $i = j$). If a tie between the old and the new trustor exists, the old trustor conveys his information about the trustee to the new trustor.[2] The probability that such a tie exists between trustors of type i and j exists is α_{ij}.[3] This probability only depends on the types of trustors and equals the density of ties between trustors of type i and j. We obtain heterogeneity, because the α_{ij} can be different between the different segments of trustors. We say there exists a path between trustors of type i and type j, if with a positive probability the information about the trustee abusing trust placed by a trustor of type i will reach trustors of type j, directly or indirectly.

In Figure 2, a network is illustrated. Trustors of type 1 are involved in half of the transactions with the trustee (π_1), trustors of type 2 in one-sixth (π_2), and the trustors of type 3 in one-third (π_3). In the example, $\alpha_{11} = 1$. Thus, ties exist between all type 1 trustors; Since $\alpha_{32} = 0$, there are no ties from trustors of type 3 to trustors of type 2. Finally, $\alpha_{12} = 1/3$ indicates that a fraction of 1/3 of all possible ties exist between trustors of type 1 and trustors of type 2.

The constituent game will be played infinitely often. If a player is not involved in a transaction at time t, he will receive a payoff equal to 0. Payoffs are depreciated exponentially with a factor w for every following game ($0 \leq w \leq 1$). These reflect pure time preferences. Thus, the total payoff associated with a stream of payoffs $(u_{i0}, u_{i1}, \ldots)$ equals $\sum_{t=0}^{\infty} w^t u_{it}$ for a player i.[4]

Finally, we have to make assumptions regarding the information that the actors have about the game. We use the extreme but standard assumption that the full description of the model and the structure of the game $\Gamma(\Gamma_\theta, F, \boldsymbol{\delta}, \boldsymbol{\pi}, \boldsymbol{A}, w)$ with all its features is common knowledge, i.e., the order of moves, the payoffs of all players etc.

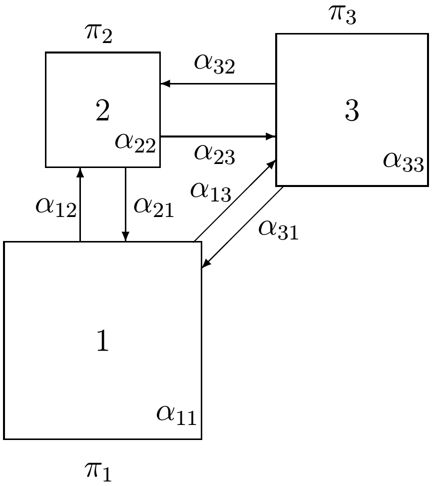
[1] To avoid notational complexity we have made the homogeneity assumption that the payoffs of the trustee and the distribution F are independent of the type of trustor. Our analyses easily generalize for the case that the payoffs depend on the type of trustor.

[2] In principal, the model can be used for symmetric and asymmetric relations. Because the concerned network ties represent mostly bilateral contacts we formulate the description in terms of symmetric relations. However, there could be some hierarchy in the types of trustors, which causes the information to go only in one direction.

[3] It is important to note that in the model is assumed that trustors do not use information they have strategically. If they have the possibility to tell somebody what they know about the trustee, they will do.

[4] We introduced only one depreciation rate, while the depreciation rate could be different for the trustee and types of trustors. Our results, however, depend only on the depreciation rate of the trustee. Therefore, if we talk about the depreciation rate (or time preferences) this is, essentially, the depreciation rate of the trustee.

Figure 2 *Example of a network of trustors.*



$$\boldsymbol{\pi} = \begin{pmatrix} 1/2 \\ 1/6 \\ 1/3 \end{pmatrix}$$

$$\boldsymbol{A} = \begin{pmatrix} 1 & 1/3 & 1/2 \\ 1/2 & 1/2 & 1/4 \\ 0 & 0 & 1 \end{pmatrix}$$

Note on heterogeneity of transactions
Contrary to most other models, our model does not assume that the payoffs are the same in every game. That is to say, the incentive θ of the trustee to abuse trust placed by the trustor, is chosen randomly from the distribution F. Again, note that the payoffs do not depend on the history of the game.

There are two reasons why it is useful to analyze an IHTG instead of an iterated homogeneous Trust Game. First, it is more realistic to assume that the incentive to abuse trust is not the same in every transaction. Second, the more complicated heterogeneous game is still mathematically tractable and results in equilibria with an interesting property: trustor and trustee cooperate, i.e., trust is placed and honored, if the incentive θ_t for opportunistic behavior is smaller than a certain threshold value ϑ. Hence, the players cooperate with a certain probability $F(\vartheta)$. The threshold ϑ, or alternatively $F(\vartheta)$, can be interpreted as a predictor for how (in)complete contracts in a certain transaction are. Furthermore, we can calculate comparative statics of this predictor for several parameters of the model. In the iterated standard Trust Game we would have found equilibria where actors cooperate or defect deterministically, which gives only possibilities to argue why there should or should not be a complete contract.

3 The Solution of the Model

To analyze the model, we start with defining strategies for the game $\Gamma(\Gamma_\theta, F, \boldsymbol{\delta}, \boldsymbol{\pi}, \boldsymbol{A}, w)$. A decision node can be characterized by the pair (i, θ), where i is the involved type of trustor and θ as described before. We will analyze subgame perfect equilibria (spe's) in *trigger strategies* (Friedman, 1971). Trigger strategies are defined via trigger thresholds ϑ_i. All trustors of type i use a trigger ϑ_{i1}.[5] In a decision node (i, θ) a trustor of type i

[5]Because trustors of the same type are completely identical they have the same trust-threshold and the trustee will use the same threshold in relation with trustors of the same type. Cf. the symmetry requirement in Harsanyi and Selten (1988, 70-71).

will play C_1 if $\theta \leq \vartheta_{i1}$ and the trustor has no information that the trustee abused trust (D_2). Otherwise, he plays D_1. Consequently, as soon as the trustor obtains information about abused trust by the trustee, either from own experience or from another trustor, he will always play D_1.

Similarly, the trustee chooses a ϑ_{i2} for a decision node (i, θ). That means that the trustee will honor trust (C_2) if $\theta \leq \vartheta_{i2}$ and abuse trust (D_2) if trust is placed by a trustor of type i for $\theta > \vartheta_{i2}$.

An important advantage of trigger strategies is that they can be analyzed in an elegant way. Moreover, we will find equilibria in which trust is only placed if the incentive for abusing trust is not too large. These equilibria are suboptimal, because if trust would always be placed and honored, all players would receive more. The trust-threshold can be interpreted as a predictor for the incompleteness of contracts in trust relations. Furthermore, it gives a measure for efficiency, because we can compare the trigger equilibria with the situation that trust is always placed and honored. The higher the trust-threshold ϑ, the higher the proportion of transaction $F(\vartheta)$ that is arranged with incomplete contracts (as long as F itself is not changed), the higher the efficiency.

The following theorem shows that in spe the thresholds for the trustee and the trustor are the same ($\vartheta_{i1} = \vartheta_{i2}$). Hence, on the equilibrium path, the actors play C_1C_2 or D_1, but never C_1D_2. In other words, trust will not always be placed, but if trust is placed, it will never be abused in equilibrium.[6].

Theorem 1 *The* IHTG $\Gamma(\Gamma_\theta, F, \boldsymbol{\delta}, \boldsymbol{\pi}, \boldsymbol{A}, w)$ *has the following properties.*

i) There exists at least one spe in trigger strategies, namely, $\vartheta_{i1} = \vartheta_{i2} = 0$ *for all i;*

ii) If a vector of trigger strategies is a spe, $\vartheta_{i1} = \vartheta_{i2}$ *for all i.*

Proof. We refer to the Appendix for the proofs of the theorems. □

As a result of this theorem, we will denote trigger strategy vectors by $\boldsymbol{\vartheta} = (\vartheta_1, \ldots, \vartheta_k)$. We will now deduce conditions for which the trigger strategies $\boldsymbol{\vartheta}$ form a spe. To analyze the consequences of a particular behavior by the players in the IHTG, we have to examine in detail what can happen between two constituent games. Two aspects are important, namely, the identity of the next trustor and the information about the behavior of the trustee that will be possibly transferred to the next trustor. It has to be kept in mind that the dying trustor may transfer his information on the trustee's behavior to the subsequent trustor only at that moment. Henceforth, a trustor is called informed if he has information on non-cooperative behavior by the trustee. Although this is not the usual notion of being informed, it is the most useful here, because a trustor who receives information about cooperative behavior of the trustee will act the same as a trustor who has no information at all. For technical details we refer to the appendix.

The next theorem states our central results. First, a condition for a spe in trigger strategies is given. For specific properties of the set of spe's in trigger strategies of our model, we refer to Theorem 3 in the Appendix. Because, we find multiple equilibria in trigger strategies, we still cannot make real predictions. Therefore, we have to choose equilibria, or preferably a unique equilibrium, that most probably will be chosen by the

[6]There exist many equilibria that involve other strategies as is the case more generally in supergame theory (see for some background and references on this result (Folktheorem): Friedman, 1986, 103-104)

players. For equilibrium selection within the class of trigger strategies we use payoff dominance (Harsanyi and Selten, 1988, 80-81). This means that we consider an equilibrium more probable than another if it is a Pareto improvement. The second part of the next theorem states that payoff dominance yields a unique spe in trigger strategies.

Theorem 2 *Consider the* IHTG $\Gamma(\Gamma_\theta, F, \boldsymbol{\delta}, \boldsymbol{\pi}, \boldsymbol{A}, w)$.

i) The vector $\boldsymbol{\vartheta} = (\vartheta_1, \ldots, \vartheta_k)$ *of trigger strategies is in spe if and only if*

$$\vartheta_i \leq (R_2 - P_2)\boldsymbol{e}_i'(\tilde{\boldsymbol{T}}_w - \boldsymbol{I})F(\boldsymbol{\vartheta}) \text{ for all } i. \tag{1}$$

ii) There exists a unique Pareto optimal spe, in the class of trigger strategies, with thresholds ϑ^* *for which*

$$\vartheta_i^* = (R_2 - P_2)\boldsymbol{e}_i'(\tilde{\boldsymbol{T}}_w - \boldsymbol{I})F(\boldsymbol{\vartheta}^*) \text{ for all } i. \tag{2}$$

Where,

$\boldsymbol{e}_i$ *is the i-th unit vector of length k;*

$\tilde{\boldsymbol{T}}_w = (\boldsymbol{I} - w\boldsymbol{T})^{-1}$;

T_{ij} *is the probability that an informed trustor of type i is followed by an informed trustor of type j;*

$F(\boldsymbol{\vartheta}) = (F(\vartheta_1), \ldots, F(\vartheta_k))$.

Although (2) seems a rather complicated formula, it can be interpreted straightforwardly. Namely, the trust-threshold equals the product of the costs of contracting for the trustee, the incentives for abusing trust, and the matrix $(\tilde{\boldsymbol{T}}_w - I)$, where the element $(\tilde{\boldsymbol{T}}_w - I)_{ij}$ is exactly the expected number of times a trustor of type j will not place trust after the trustee has abused trust placed by a trustor of type i.

Now, we can consider some special cases for which the complexity of the inequality (2) diminishes considerably. First, without any social network effects and no temporal embeddedness, ($\alpha_{ij} = 0$ and $\delta_i = 1$ for all i and j), we are back in the one-shot game. No information of abusing trust by the trustee is transferred to the next game. Thus, the only equilibrium is the equilibrium in which trust is never placed, $\boldsymbol{\vartheta} = \boldsymbol{0}$. Second, if only $\alpha_{ij} = 0$ for all i and j, no information transfer between different trustors occurs. The extent to which trust is placed should satisfy

$$\vartheta_i = \frac{w(1-\delta_i)}{1 - w(1-\delta_i)}(R_2 - P_2)F(\vartheta_i) \text{ for all i.} \tag{3}$$

Thus, the trust-thresholds are higher if temporal embeddedness increases, (δ_i decreases), if the depreciation rate is higher (w) and if the costs of contracting are higher ($R_2 - P_2$).

Third, if $\alpha_{ij} = 1$ for all i and j, information of abusing trust by the trustee will be known for ever by the trustors who have to play the game. Therefore, trust will never be placed after the first defection of the trustee. Thus, the δ's do not matter anymore. The restriction for spe in this case is given by

$$\vartheta_i = \frac{w}{1-w}(R_2 - P_2)F(\vartheta_i) \text{ for all } i. \tag{4}$$

Finally, if $\alpha_{ij} = \alpha$ and $\delta_i = \delta$ for all i and j, equivalent to $k = 1$, we obtain, a result known from Weesie et al. (1995),

$$\vartheta = \frac{w(1-\delta(1-\alpha))}{1-w(1-\delta(1-\alpha))}(R_2 - P_2)F(\vartheta). \tag{5}$$

This implies that the trust-thresholds increase in the network density α.

We will consider the payoff dominant spe given in Theorem 2 to be the solution of the IHTG. Consequently, $F(\boldsymbol{\vartheta}^*)$ will serve as our predictor for the behavior of the trustor, i.e., the extent to which a trustor uses complete contracts. We are especially interested in how this predictor depends on the parameters of the model. For the special cases we have already given some results. The next theorem states how the solution of the IHTG depends on the parameters.

Theorem 4 *The payoff dominant spe in trigger strategies with thresholds $\boldsymbol{\vartheta}^*$ has the following properties.*

i) ϑ_i^* and $F(\vartheta_i^*)$ increase in the costs of contracts $R_2 - P_2$ for all i.

ii) ϑ_i^* and $F(\vartheta_i^*)$ increase in the time preference w of the trustee for all i and is independent of the time preferences of the trustors.

iii) ϑ_i^* and $F(\vartheta_i^*)$ decrease in the death rate δ_j if and only if in the type network associated with $\boldsymbol{A}$ a path exists from trustors of type i to trustors of type j.

iv) ϑ_i^* and $F(\vartheta_i^*)$ increase in T_{jk} and α_{jk} if and only if in the type network associated with $\boldsymbol{A}$ a path exists from trustors of type i to trustors of type j.

v) ϑ_i^* decreases in F in the sense of stochastic ordering; thus if $F_1(\theta) > F_2(\theta)$ for all θ then $\boldsymbol{\vartheta}^*(F_1) > \boldsymbol{\vartheta}^*(F_2)$.

By Theorem 4, the trustor demands less complete contracts and efficiency will be higher if the difference between payoffs for mutual defection and mutual cooperation for the trustee $(R_2 - P_2)$ is larger. In other words, if the costs of complete contracting to the trustee are higher, trustors will more frequently rely on incomplete contracts. Moreover, if the future is more important to the trustee (w is larger), punishment by the trustor will be more severe for the trustee and so the trust-threshold ϑ_i^* and the efficiency $F(\vartheta_i^*)$ will become higher. If the incentives for abusing trust become smaller (in the sense of stochastic ordering), less complete contracts are needed. If the trustee deals with the same trustor for a longer time (δ_i is smaller), trustors of type i have better punishment possibilities and so ϑ_i^* becomes larger. In addition, who, directly or indirectly, have ties to trustors of type i can increase there trust-thresholds.

Finally, we have seen above that the reliance on incomplete contracts and, hence, efficiency increases in the density of the networks. In addition, if any density α_{ij} of a network between two types of trustors becomes higher, this has positive consequences for the trustors who are directly or indirectly connected with this tie. Essentially, both 'health' and social networks act as public goods from which everybody can profit.

4 Network Implications

From the foregoing sections, we know how to find a predictor for incompleteness of contracts between a trustor and trustee. However, we do not have any idea about network structures that are especially efficient, except that more information is 'better'. In this section we want to address a problem of 'social engineering'. We assume that the number of ties a trustor has to transfer his information to others is given. How should this trustor arrange his ties to optimize the level of his trust-threshold? The number of ties from somebody to somebody else is called the *outdegree*. In our model, however, we do not use the absolute number of ties, but the percentage of people somebody is connected to. Therefore, the *outdegree* α_i of a trustor of type i is defined as the probability that he can transfer his information to the next trustor,

$$\alpha_i = \sum_{j=1}^{k} \pi_j \alpha_{ij}. \tag{6}$$

In the following theorems, one essential feature that determines the trust-threshold of a trustor of type i is the expected time information of an possible abuse of trust by the trustee will stay in the network of trustors. This process is equivalent to a Markov process with $2k$ states obtained as the Cartesian product of the set of types, $1 \ldots k$, and an indicator whether or not the player is informed. We assume that at a certain moment information is introduced into the network (the trustee abused trust), and calculate the expected time until the information about the abuse of trust has left the network. From the theory of Markov chains (e.g., Bradley and Meek, 1986), we know that the depreciated expected number of times γ_i trustors know about the abuse of trust placed by a trustor of type i, satisfies

$$\gamma_i = \sum_{j=1}^{k} (\tilde{\boldsymbol{T}}_w - \boldsymbol{I})_{ij}. \tag{7}$$

We will call γ_i also the *chain length*. The following theorem states some first results of the effects of network properties on ϑ_i^*.

Theorem 5 *The Pareto optimal trust-thresholds $\boldsymbol{\vartheta}$ in trigger strategies have the following properties.*

i) If for all i, $\delta_i = \delta$ and $\alpha_i = \alpha$,

$$\vartheta_i^* = (R_2 - P_2) \frac{(1 - \delta(1 - \alpha))w}{1 - (1 - \delta(1 - \alpha))w} F(\vartheta_i^*). \tag{8}$$

ii) If $k = 2$ and $\gamma_1 > \gamma_2$, $\vartheta_1 > \vartheta_2$.

iii) If $k > 2$, the ϑ_i^*'s are not necessarily increasing in γ_i.

By Theorem 5, we see that if all death rates δ_i and all outdegrees α_i are the same, then all trust-thresholds are the same and this, in fact, equals the trust-threshold for the special case $k = 1$. Consequently, we conclude that if all outdegrees are the same, the network structure has no influence on the trust-thresholds $\boldsymbol{\vartheta}$. Therefore, in the next theorems, in which we study network structures with given outdegrees, we can assume that these outdegrees are different. We expected that if information about abuse of trust by the trustee is in the network of the trustors for a longer time, the trust-threshold would be larger, because the consequences for the trustee are worse. However, Theorem 5*iii*) shows that if the expected number of transaction that trustors will not place trust after the seller has abused trust increases, the trust-threshold not necessarily increases. The reason for this is that not only the expected time is important but the expected time in combination with which trustors are involved. Nevertheless, in numerical simulation that we conducted, the correlation between ϑ_i^* and γ_i was always very high and examples as given in the proof of Theorem 5*iii*) are exceptions.

Now, we present a theorem for $k = 2$ to find indications about structures that lead to high levels of trust. Thereafter, we generalize this theorem with an inductive argument to $k > 2$.

Theorem 6 *Consider the* IHTG $\Gamma(\Gamma_\theta, F, \boldsymbol{\delta}, \boldsymbol{\pi}, \boldsymbol{A}, w)$ *with* $k = 2$ *and* $\delta_1 = \delta_2 = \delta$.

i) If $\alpha_1 > \alpha_2$ *then the* ϑ_1^* *and* ϑ_2^* *are optimal, with relation to the network structure, for a transition matrix* $\boldsymbol{T}$*,*

$$\boldsymbol{T} = \begin{pmatrix} 1 - \delta + \delta \min(\pi_1, \alpha_1) & \delta \max(0, \alpha_1 - \pi_1) \\ \delta \min(\pi_1, \alpha_2) & 1 - \delta + \delta \max(0, \alpha_2 - \pi_1) \end{pmatrix}. \tag{9}$$

ii) If $\alpha_1 > \alpha_2$ *and* $\alpha_{12} = \alpha_{21}$ *(symmetric relations) and we assume that bilateral relations cannot be forced unilaterally, then* ϑ_1^* *and* ϑ_2^* *are optimal, with relation to the network structure, for a transition matrix* $\boldsymbol{T}$*,*

$$\boldsymbol{T} = \begin{pmatrix} 1 - \delta + \delta \min(\pi_1, \alpha_1) & \delta \max(0, \alpha_1 - \pi_1) \\ \delta \max(0, \alpha_1 - \pi_1) & 1 - \delta + \delta(\alpha_2 - \max(0, \alpha_1 - \pi_1) \end{pmatrix}. \tag{10}$$

Theorem 6 shows that the network that is *centralized* around the trustors with the highest outdegree is the best network that can be constructed for fixed outdegrees. Part *ii*) of the Theorem considers only symmetric relations ($\alpha_{ij} = \alpha_{ji}$). Because trustors of type 1 have the most ties, the optimal distribution of ties as given in the Theorem prescribes that there are as much as possible ties between trustors of type 1. If the network between trustors of type 1 is complete and trustors of type 1 have ties left, these are ties between trustors of type 1 and trustors of type 2. The remainder of the ties of trustors of type 2 are used between themselves.

The following and final theorem generalizes the results from Theorem 6.

Theorem 7 *Consider the* IHTG $\Gamma(\Gamma_\theta, F, \boldsymbol{\delta}, \boldsymbol{\pi}, \boldsymbol{A}, w)$ *with* $\delta_1 = \delta_2 = \ldots = \delta_k = \delta$*. Then holds,*

i) If $\alpha_1 > \alpha_2 > \ldots > \alpha_k$ *then the* $\vartheta_1^*, \ldots, \vartheta_k^*$ *are maximal if the transition matrix* $\boldsymbol{T}$ *is chosen such that everybody talks as much as possible to trustors of type 1, after that talk as much as possible to trustors of type 2, etc.*

ii) If $\alpha_1 > \alpha_2 > \ldots > \alpha_k$ *and* $\alpha_{ij} = \alpha_{ji}, i \neq j$ *(symmetric relations) and we assume that bilateral relations cannot be forced unilateral, then the* ϑ_i^* *are optimal if in the transition matrix* $\boldsymbol{T}$ α_{11} *is chosen maximal first. After that choosing the* α_{ij}*'s for which* $i + j = 3$ *maximal. Then the* α_{ij}*'s for which* $i + j = 4$ *etc.*

The last two theorems do not imply that all centralized networks are better than other networks. Only networks centralized around trustors with the highest outdegrees are especially efficient in terms of our model.

5 Conclusions and Discussion

The results of the model discussed in this paper support and extend theoretical hypotheses that can be made for the management of exchange relations. In accordance with existing literature (Granovetter, 1985; Coleman 1990; Raub and Weesie, 1993; Weesie et al. 1995), less complete contracts are necessary if the contract costs for the trustee are higher ($R_2 - P_2$), if the trustee is more patient (i.e., w is higher), if the trustor expects to be involved with the trustee for a longer time (δ_i), if the average incentive for the trustee to abuse trust (F) becomes smaller, and if the density of the network of trustors becomes higher. In addition we predict, even more specifically, that if 'individual network ties' (α_{ij}) become more tight, the use of complete contracts will be less.

Complementary to results that were known from former studies, we have shown some results for specific network structures. First, if all trustors have the same outdegree, the network structure has no influence on the possibility to arrange transactions with incomplete contracts. Second, if not all outdegrees are the same, the most efficient network is the network that is centralized around the trustors with the highest outdegrees. Thus, we expect that with increasing centralization around the most contacted trustors, the use of incomplete contracts increases.

Some restrictions of our model we want to relax in future models. The most important restriction of our model is that trustors act only successively and not simultaneously. Only after a trustor dies and a new trustor is chosen, trustors can exchange information. To model parallel operating trustors there are different possibilities. In future studies, we are planning to present a stochastic information diffusion model, which will not be concerned with game-theoretical considerations. Another possibility to model parallel trustors is in a simulation study.

Another important disadvantage of our model is that we cannot make any predictions about the influence of the payoffs and time preferences of the trustors on their behavior. One reason for this is that the trustor can perfectly observe the behavior of the trustee. If this would not be the case, trustors have to be more careful with their punishment strategy, because they maybe think that the trustee defected, but in fact the trustee did not. Such punishment is also costly for the trustor and the more costly it is the more careful he should be. The trustor has to search for an optimal punishment strategy. Therefore, we will model these so-called monitoring problems for the trustor in a future model to find prediction for the relation between costs of contracts for the trustor and the use of incomplete contracts.

Appendix

In this appendix, we prove the theorems presented in the paper and give some technical details.

Proof of Theorem 1. If all players play D_i all the time, this constitutes a spe. This proves assertion i). Now consider a strategy vector $(\vartheta_{11}, \ldots, \vartheta_{k1}, \vartheta_{12}, \ldots, \vartheta_{k2})$. On the one hand, assume $\vartheta_{i1} < \vartheta_{i2}$ for at least one i. Then, the trustors of type i do not maximize their payoffs, because if they take $\vartheta_{i1} = \vartheta_{i2}$ their payoff increases with $R_1 - P_1$ every time that $\vartheta_{i1} < \theta < \vartheta_{i2}$. This occurs with positive probability since F has full support. In all other cases the payoff is the same. On the other hand, assume $\vartheta_{i1} > \vartheta_{i2}$ for at least one i. In this case, trustors of type i do not maximize their payoffs, because in the situation that $\vartheta_{i1} > \theta > \vartheta_{i2}$ the trustee will play D_2 and the trustor involved C_1. The trustee will receive $R_2 + \theta$, while the trustor receives S and in all the following games the trustor plays D_1 and receives a payoff P_1. However, if the trustor in this case had chosen $\vartheta_{i1} = \vartheta_{i2}$, he would have received P_1 in the game mentioned and R_1 or P_1 in all the following games, which is more than he receives now. Thus, if $\vartheta_{i1} \neq \vartheta_{i2}$ the trustors of type i increase their payoffs by moving their threshold towards the threshold of the trustee. Therefore, if $\vartheta_{i1} \neq \vartheta_{i2}$ the trustor is not using a best reply. □

For the remainder of the appendix it will be useful to calculate the transitional probabilities between the different states. After an interaction of a trustor of type i with the trustee, we have four possibilities for the following interaction.

1. The previous interaction involved the same trustor of type i as the next transaction. This happens with probability $1 - \delta_i$.
2. The previous interaction involved an informed trustor of type i, he dies and the next transaction is with an informed trustor of type j. This happens with probability $\delta_i \pi_j \alpha_{ij}$. Here, δ_i is the probability that a trustor of type i dies; π_j is the probability that a trustor of type j is chosen; α_{ij} is the probability that information the trustor of type i has is transferred to a trustor of type j; note that possibly $i = j$.
3. The previous interaction involved an informed trustor of type i, he dies and the next transaction is with an uninformed trustor of type j. This happens with probability $\delta_i \pi_j (1 - \alpha_{ij})$.
4. The previous interaction involved an uninformed trustor of type i, he dies and the next transaction is with an uninformed trustor of type j. The probability that this happens equals $\delta_i \pi_j$.

Note that it is impossible that an uninformed trustor dies and for the next game an informed trustor is chosen.

Define the states I_i and $\overline{I}_i$ which denote a trustor i who is informed and a trustor i who is not. We define a transition matrix $\boldsymbol{Q}$ which gives the probabilities for the different transition possibilities (rows are 'origins' and columns are 'destinations'),

$$\boldsymbol{Q} = \begin{array}{c} \\ I \\ \overline{I} \end{array} \begin{array}{c} \begin{array}{cc} I & \overline{I} \end{array} \\ \left(\begin{array}{c|c} \boldsymbol{T} & \boldsymbol{\Pi} - \boldsymbol{T} \\ \hline \boldsymbol{0} & \boldsymbol{\Pi} \end{array} \right) \end{array}. \tag{11}$$

Here $\boldsymbol{T}$ denotes the probabilities for transitions between informed states and $\boldsymbol{\Pi}$ the probabilities for transitions between uninformed states. The expressions for $\boldsymbol{T}$ and $\boldsymbol{\Pi}$ follow from the definitions above.

$$\boldsymbol{T} = \begin{pmatrix} 1 - \delta_1 + \delta_1 \pi_1 \alpha_{11} & \cdots & \delta_1 \pi_k \alpha_{1k} \\ \vdots & \ddots & \vdots \\ \delta_k \pi_1 \alpha_{k1} & \cdots & 1 - \delta_k + \delta_k \pi_k \alpha_{kk} \end{pmatrix}, \text{, and} \tag{12}$$

$$\mathbf{\Pi} = \begin{pmatrix} 1-\delta_1+\delta_1\pi_1, & \cdots & \delta_1\pi_k \\ \vdots & \ddots & \vdots \\ \delta_k\pi_1, & \cdots & 1-\delta_k+\delta_k\pi_k \end{pmatrix}. \tag{13}$$

Henceforth, $\boldsymbol{\mu}$ is a k-vector with $\mu_i = F(\vartheta_i)R_2 + (1-F(\vartheta_i))P_2$; $\boldsymbol{p}$ is an all P_2 k-vector. A matrix of the form $(\boldsymbol{I} - w\boldsymbol{M})^{-1}$, where $\boldsymbol{M}$ can be any matrix, is denoted as $\tilde{\boldsymbol{M}}_w$. Note that $\boldsymbol{M}\tilde{\boldsymbol{M}}_w = \tilde{\boldsymbol{M}}_w - \boldsymbol{I}$. Because the sum of the rows in the matrix $w\boldsymbol{M}$ is always smaller than one and all elements of $w\boldsymbol{M}$ are positive, $\boldsymbol{I} - w\boldsymbol{M}$ is regular, i.e., $\boldsymbol{I} - w\boldsymbol{M}^{-1}$ exists (Berman and Plemmons 1979, 133).

Proof of Theorem 2. First, we will prove the equilibrium condition of assertion i). According to a well-known result of dynamic programming theory, Bellman's optimality principle (Kreps, 1990a), we know that optimality on the total path is guaranteed if deviation from the prescribed path in any one decision node does not increase the payoff. Therefore, we have to prove that if a player makes a one-step deviation from the equilibrium path, his payoff will decrease. Without loss of generality, we look at deviations in the first step. The involved trustor is of type i. First, consider $\theta > \vartheta_i$. Here, both player defect, and therefore, no one has an incentive to deviate. Now, consider $\theta \leq \vartheta_i$. Again the trustor has no incentive to deviate, because on the equilibrium path he receives his maximal payoff R_1. The trustee should play D_2 if he can obtain a short term profit that is higher than the long term punishment by the trustors. We have to prove that the restriction in Theorem 2 is exactly the condition that the long term punishment will be larger than the short term profit for the trustee, if he plays D_2 and $\theta \leq \vartheta_i$. Thus, that the trustee also has no incentive to shift from the equilibrium path. Because this holds for all θ and all i, the trigger strategies are in spe.

Let $EU_2(C_2, \theta; \boldsymbol{\vartheta})$ be the payoff for the trustee if both follow the trigger strategy. Here, no trustor is ever informed on deviations from the equilibrium path by the trustee. Therefore, the probabilities for the following trustors are given in the matrix $\mathbf{\Pi}$. The trustee's payoff equals P_2 with probability $Pr(\theta > \vartheta_i) = 1 - F(\vartheta_i)$ and R_2 with probability $F(\vartheta_i)$. The trustee's expected payoff for the equilibrium path equals

$$EU_2(C_2, \theta; \boldsymbol{\vartheta}) = R_2 + \sum_{t=1}^{\infty} w^t \mathbf{\Pi}^t \boldsymbol{\mu} = R_2 + \boldsymbol{e}_i'(\tilde{\mathbf{\Pi}}_w - \boldsymbol{I})\boldsymbol{\mu}. \tag{14}$$

Now the trustee deviates from the equilibrium path and abuses placed trust (D_2). The necessary condition for spe is that $EU_2(C_2, \theta; \boldsymbol{\vartheta}) \geq EU_2(D_2, \theta; \boldsymbol{\vartheta})$ for all $\theta \leq \vartheta_i$. The involved trustor is informed. Therefore, the probabilities whether or not the following trustors are informed, are given in the matrix $\boldsymbol{Q}$. The payoff for the trustee is $R_2 + \theta$ in the first step; P_2 as long as an informed trustor is involved. As soon as an uninformed trustor comes, in the payoff is as given before. Thus,

$$\begin{aligned} EU_2(D_2, \theta; \boldsymbol{\vartheta}) &= R_2 + \theta + \sum_{t=1}^{\infty} w^t \begin{pmatrix} \boldsymbol{e}_i' & \mathbf{0}' \end{pmatrix} \boldsymbol{Q}^t \begin{pmatrix} \boldsymbol{p} \\ \boldsymbol{\mu} \end{pmatrix} \\ &= R_2 + \theta + \begin{pmatrix} \boldsymbol{e}_i' & \mathbf{0}' \end{pmatrix} (\tilde{\boldsymbol{Q}}_w - \boldsymbol{I}) \begin{pmatrix} \boldsymbol{p} \\ \boldsymbol{\mu} \end{pmatrix}. \end{aligned} \tag{15}$$

By straightforward computation,

$$\tilde{\boldsymbol{Q}}_w - \boldsymbol{I} = \begin{pmatrix} \boldsymbol{I} - w\boldsymbol{T} & -w(\mathbf{\Pi} - \boldsymbol{T}) \\ \mathbf{0} & \boldsymbol{I} - w\mathbf{\Pi} \end{pmatrix}^{-1} - \boldsymbol{I}$$

$$
\begin{aligned}
&= \begin{pmatrix} \tilde{\boldsymbol{T}}_w & w\tilde{\boldsymbol{T}}_w(\boldsymbol{\Pi}-\boldsymbol{T})\tilde{\boldsymbol{\Pi}}_w \\ \mathbf{0} & \tilde{\boldsymbol{\Pi}}_w \end{pmatrix} - \boldsymbol{I} \\
&= \begin{pmatrix} \tilde{\boldsymbol{T}}_w & \tilde{\boldsymbol{\Pi}}_w - \tilde{\boldsymbol{T}}_w \\ \mathbf{0} & \boldsymbol{I} + \frac{w}{1-w}\boldsymbol{\Pi} \end{pmatrix} - \boldsymbol{I} \\
&= \begin{pmatrix} \tilde{\boldsymbol{T}}_w - \boldsymbol{I} & \tilde{\boldsymbol{\Pi}}_w - \tilde{\boldsymbol{T}}_w \\ \mathbf{0} & \frac{w}{1-w}\boldsymbol{\Pi} \end{pmatrix}.
\end{aligned} \tag{16}
$$

By substituting (16) in (15):

$$
\begin{aligned}
EU_2(D_2,\theta;\boldsymbol{\vartheta}) &= R_2 + \theta + \boldsymbol{e}_i'(\tilde{\boldsymbol{T}}_w - \boldsymbol{I})\boldsymbol{p} + \boldsymbol{e}_i'(\tilde{\boldsymbol{\Pi}}_w - \tilde{\boldsymbol{T}}_w)\boldsymbol{\mu} \\
&= R_2 + \theta + \boldsymbol{e}_i'(\tilde{\boldsymbol{T}}_w - \boldsymbol{I})(\boldsymbol{p} - \boldsymbol{\mu}) + \boldsymbol{e}_i'(\tilde{\boldsymbol{\Pi}}_w - \boldsymbol{I})\boldsymbol{\mu}.
\end{aligned} \tag{17}
$$

Hence, $EU_2(C_2,\theta;\boldsymbol{\vartheta}) \geq EU_2(D_2,\theta;\boldsymbol{\vartheta})$ is equivalent to

$$
\begin{aligned}
\theta &\leq \boldsymbol{e}_i'(\tilde{\boldsymbol{T}}_w - \boldsymbol{I})(\boldsymbol{\mu} - \boldsymbol{p}) \\
&= \boldsymbol{e}_i'(\tilde{\boldsymbol{T}}_w - \boldsymbol{I}) \begin{pmatrix} (R_2 - P_2)F(\vartheta_1) \\ \vdots \\ (R_2 - P_2)F(\vartheta_k) \end{pmatrix} \\
&= (R_2 - P_2)\boldsymbol{e}_i'(\tilde{\boldsymbol{T}}_w - \boldsymbol{I})F(\boldsymbol{\vartheta}).
\end{aligned} \tag{18}
$$

In equilibrium (18) should hold for all $\theta \leq \vartheta_i$: $EU_2(C_2,\theta;\boldsymbol{\vartheta}) \geq EU_2(D_2,\theta;\boldsymbol{\vartheta})$; clearly $\theta = \vartheta_i$ is the most restrictive, which implies $\vartheta_i \leq (R_2 - P_2)\boldsymbol{e}_i'(\tilde{\boldsymbol{T}}_w - \boldsymbol{I})F(\boldsymbol{\vartheta})$ for all i. This finishes the proof of the equilibrium condition.

Now we prove the existence of a unique Pareto optimal equilibrium in trigger strategies. Suppose there exists a spe with $\vartheta_i < (R_2 - P_2)\sum_{j=1}^k (\tilde{\boldsymbol{T}}_w - \boldsymbol{I})_{ij}F(\vartheta_j)$ for at least one i. Then ϑ_i can be increased until $\hat{\vartheta}_i = (R_2 - P_2)\sum_{j=1}^k (\tilde{\boldsymbol{T}}_w - \boldsymbol{I})_{ij}F(\vartheta_j)$ because the right side is bounded above by γ, while ϑ_i is non bounded above. All the other inequalities still hold because the right side increases with ϑ_i. Thereafter, we have to check whether $\vartheta_{i+1} = (R_2 - P_2)\sum_{j=1}^k (\tilde{\boldsymbol{T}}_w - \boldsymbol{I})_{i+1j}F(\vartheta_j)$, otherwise ϑ_{i+1} can be increased until a $\hat{\vartheta}_{i+1} > \vartheta_{i+1}$. Continue this procedure until ϑ_k and start again from ϑ_1, etc. This gives an increasing series of vectors $\boldsymbol{\vartheta}$, which is bounded because $\vartheta_i \leq \gamma$. By Weierstrass' Theorem, the series converges to a limit where $\vartheta_i = (R_2 - P_2)\sum_{j=1}^k (\tilde{\boldsymbol{T}}_w - \boldsymbol{I})_{ij}F(\vartheta_j)$ for all i and this solution is a Pareto improvement of all equilibria found before.

We show now that there exists a unique $\boldsymbol{\vartheta}^*$ that Pareto dominates all other equilibria. Suppose that there are two equilibria $\boldsymbol{\vartheta}$ and $\hat{\boldsymbol{\vartheta}}$ that do not Pareto dominate each other, then there exists, after relabeling, an l such that

$$
\vartheta_i \geq \hat{\vartheta}_i \text{ for } 1 \leq i \leq l \text{ and } \vartheta_i < \hat{\vartheta}_i \text{ for } l < i \leq k \tag{19}
$$

Define $G_i(\boldsymbol{\vartheta}) = \vartheta_i - (R_2 - P_2)\sum_{j=1}^k (\tilde{\boldsymbol{T}}_w - \boldsymbol{I})_{ij}F(\vartheta_j)$. We see immediately that $\partial G_i/\partial\vartheta_j < 0$ if $i \neq j$. Then,

$$
G_i(\vartheta_1,\ldots,\vartheta_l,\hat{\vartheta}_{l+1},\ldots\hat{\vartheta}_k) < G_i(\vartheta_1,\ldots,\vartheta_k) = 0, \qquad 1 \leq i \leq l; \tag{20}
$$

$$
G_i(\vartheta_1,\ldots,\vartheta_l,\hat{\vartheta}_{l+1},\ldots\hat{\vartheta}_k) \leq G_i(\hat{\vartheta}_1,\ldots,\hat{\vartheta}_k) = 0, \qquad l < i \leq k. \tag{21}
$$

Therefore, $(\vartheta_1,\ldots,\vartheta_l,\hat{\vartheta}_{l+1},\ldots\hat{\vartheta}_k)$ is a spe. From this equilibrium we can find a Pareto improvement of the equilibria we had before, following the method of the first part of the proof. Thus, if we have two Pareto non-comparable equilibria, we have constructed a spe that is a Pareto improvement of the two equilibria found before. □

By Theorem 2, we have found a set of spe's that has the form

$$\Theta = \{\boldsymbol{\vartheta} | 0 \leq \vartheta_i \leq (R_2 - P_2)\boldsymbol{e}_i(\tilde{\boldsymbol{T}}_w - \boldsymbol{I})F(\boldsymbol{\vartheta}) \text{ for all } i\} \tag{22}$$

Theorem 3 *The set Θ, defined in (22), has the following properties.*

i) The set is not empty, because $\mathbf{0} \in \Theta$.

ii) $\Theta \subset [0, \gamma]^k$ where $\gamma = \frac{w(R_2 - P_2)}{1-w}$.

iii) The set Θ is closed.

iv) A sufficient condition for the existence of another spe than $\mathbf{0}$ is that there exists an $S \subset \{1, \ldots, k\}$, for which

$$(R_2 - P_2)F'(0) \sum_{j \in S} (\tilde{\boldsymbol{T}}_w - \boldsymbol{I})_{ij} > 1 \text{ for all } i \in S. \tag{23}$$

v) If F is concave, Θ is a convex set.

Proof. Assertion i) is already shown in Theorem 1. To prove ii), if $\theta \geq \frac{w}{1-w}(R_2 - P_2)$ the incentive for abusing trust in that particular game is larger than the maximal amount that can be earned by cooperative behavior in all the following games. Assertion iii) holds because F is continuous. To prove iv), assume that (23) holds for some S. Then, a $\varepsilon > 0$ exists, such that for all i

$$\varepsilon < (R_2 - P_2)(F(\varepsilon) - F(0)) \sum_{j \in S} (\tilde{\boldsymbol{T}}_w - \boldsymbol{I})_{ij} = (R_2 - P_2)F(\varepsilon) \sum_{j \in S} (\tilde{\boldsymbol{T}}_w - \boldsymbol{I})_{ij}. \tag{24}$$

Therefore, $\boldsymbol{\vartheta}$ with $\vartheta_j = \varepsilon$ if $j \in S$ and $\vartheta_j = 0$ if $j \notin S$ is a spe. The fact that Θ is convex follows from substitution of a convex combination of two equilibria in the inequalities.

□

Proof of Theorem 4. The iterative algorithmic argument in the proof of Theorem 2 implies that if for a certain change in the parameters, a ϑ_i can be increased, we can find a new spe that is a Pareto improvement of the equilibria found before. Therefore, for comparative statics we have to study the following expression.

$$\vartheta_i^* = (R_2 - P_2) \sum_{j=1}^{k} (\tilde{\boldsymbol{T}}_w - I)_{ij} F(\vartheta_j^*). \tag{25}$$

Thus, if the righthand side of the given expression increases for at least one i, while we do not affect any ϑ_j^*, we can increase ϑ_i^* to find a new spe and we will find a Pareto improved equilibrium from that.

Therefore, define $H_i = (R_2 - P_2) \sum_{j=1}^{k} (\tilde{\boldsymbol{T}}_w - I)_{ij} F(\vartheta_j)$. To prove i), note that $F(\theta) \geq 0$ and all matrix elements of $\tilde{\boldsymbol{T}}_w - \boldsymbol{I}$ are positive, so $\frac{\partial H_i}{\partial (R_2 - P_2)} > 0$. Assertion ii) follows because $\frac{\partial \tilde{\boldsymbol{T}}_w}{\partial w} = -\tilde{\boldsymbol{T}}_w \frac{\partial (\boldsymbol{I} - w\boldsymbol{T})}{\partial w} \tilde{\boldsymbol{T}}_w > 0$ and therefore $\frac{\partial H_i}{\partial w} > 0$. To prove iii), note that if $(\tilde{\boldsymbol{T}}_w)_{ij} > 0$ then $\frac{\partial \sum_{l=1}^{k} (\tilde{\boldsymbol{T}}_w)_{il}}{\partial \delta_j} > 0$ and therefore $\frac{\partial H_i}{\partial \delta_j} > 0$. This is exactly the case if a path exists from trustors of type i to trustors of type j. The argument for iv) is similar to the argument of iii). It holds that if $(\tilde{\boldsymbol{T}}_w)_{ij} > 0$ then $\frac{\partial \sum_{l=1}^{k} (\tilde{\boldsymbol{T}}_w)_{il}}{\partial \alpha_{jk}} > 0$ and therefore $\frac{\partial H_i}{\partial \alpha_{jk}} > 0$. To prove v), note that $R_2 - P_2 > 0$ and all matrix elements are positive, H_i increases for all i if we change from F_1 to F_2. □

Proof of Theorem 5. To prove i), note that in every period the probability that the trustor in the following period has information about the behavior of the trustee in the last period is the same. Therefore,

$$\sum_{j=1}^{k} (\tilde{\boldsymbol{T}}_w - I)_{ij} = \frac{w(1-\delta(1-\alpha))}{1-w(1-\delta(1-\alpha))} \text{ for all } i. \tag{26}$$

Thus, $\boldsymbol{\vartheta} = (\vartheta, \ldots, \vartheta)$ is a spe if and only if

$$\vartheta \leq (R_2 - P_2) \frac{w(1-\delta(1-\alpha))}{1-w(1-\delta(1-\alpha))} F(\vartheta). \tag{27}$$

Furthermore, if $\boldsymbol{\vartheta}$ is not the Pareto optimal spe, we can increase all ϑ_i with the same amount to reach the Pareto optimum in which equalities will hold.

For assertion ii), assume $\vartheta_1 \leq \vartheta_2$. Then the following inequalities hold.

$$\begin{aligned} \vartheta_2 &= (R_2 - P_2) \sum_{j=1}^{2} (\tilde{\boldsymbol{T}} - \boldsymbol{I})_{2j} F(\vartheta_j) \\ &\leq (R_2 - P_2) F(\vartheta_2) \sum_{j=1}^{2} (\tilde{\boldsymbol{T}} - \boldsymbol{I})_{2j} \\ &< (R_2 - P_2) F(\vartheta_2) \sum_{j=1}^{2} (\tilde{\boldsymbol{T}} - \boldsymbol{I})_{1j}. \end{aligned} \tag{28}$$

Therefore, $(\vartheta_2, \vartheta_2)$ is a feasible solution of the inequalities of Theorem 2 and from Theorem 2 we know that there is a spe that is a Pareto improvement of the equilibrium $(\vartheta_2, \vartheta_2)$, which is already a Pareto improvement of $(\vartheta_1, \vartheta_2)$ and it still can be improved because (28) is a strict inequality.

To prove iii), we give an example. Choose $k = 3$, $w = 0.9$, $R_2 - P_2 = 2$, $F(\theta) = \frac{\theta}{1+\theta}$ and

$$\boldsymbol{T} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0.20 & 0.25 & 0.25 \end{pmatrix}. \tag{29}$$

Then,

$$\begin{pmatrix} \sum_{j=1}^{3} (\tilde{\boldsymbol{T}} - \boldsymbol{I})_{1j} \\ \sum_{j=1}^{3} (\tilde{\boldsymbol{T}} - \boldsymbol{I})_{2j} \\ \sum_{j=1}^{3} (\tilde{\boldsymbol{T}} - \boldsymbol{I})_{3j} \end{pmatrix} = \begin{pmatrix} 9 \\ 4.3 \\ 4.1 \end{pmatrix} \text{ while } \begin{pmatrix} \vartheta_1^* \\ \vartheta_2^* \\ \vartheta_3^* \end{pmatrix} = \begin{pmatrix} 17 \\ 7.5 \\ 7.6 \end{pmatrix}. \tag{30}$$

□

Proof of Theorem 6. To prove i), define in the transition matrix $\boldsymbol{T}$ $T_i = 1 - \delta + \delta\alpha_i$. Then,

$$\boldsymbol{T} = \begin{pmatrix} T_{11} & T_1 - T_{11} \\ T_2 - T_{22} & T_{22} \end{pmatrix}. \tag{31}$$

Assuming that we have a spe for given T_{11} and T_{22}, we know from Theorem $5ii$) that $F(\vartheta_1) > F(\vartheta_2)$, because by calculating the matrix $(\tilde{\boldsymbol{T}}-\boldsymbol{I})$ it can be seen immediately that $\sum_{j=1}^{2}(\tilde{\boldsymbol{T}}-\boldsymbol{I})_{1j} > \sum_{j=1}^{2}(\tilde{\boldsymbol{T}}-\boldsymbol{I})_{2j}$. It follows from straightforward calculations that $\frac{\partial H_i}{\partial T_{11}} > 0$ for $i = 1, 2$ and $\frac{\partial H_i}{\partial T_{22}} < 0$ for $i = 1, 2$. That means that we can find a spe that is a Pareto improvement of the initial equilibrium by making T_{11} as large as possible and T_{22} as small as possible, subject to $T_{ii} \leq 1 - \delta + \delta\pi_i$ and $T_{ij} \leq \delta\pi_j, i \neq j$. The solution of the constrained optimization process is provided in i).

To prove ii), define in the transition matrix $\boldsymbol{T}$ $T_i = 1 - \delta + \delta\alpha_i$. Thus,

$$\boldsymbol{T} = \begin{pmatrix} T_{11} & T_1 - T_{11} \\ T_1 - T_{11} & T_2 - T_1 + T_{11} \end{pmatrix}. \tag{32}$$

Assuming that we have a spe for given T_{11}, we know from Theorem $5ii$) that $F(\vartheta_1) > F(\vartheta_2)$. It follows from straightforward calculations that $\frac{\partial H_1}{\partial T_{11}} > 0$, but $\frac{\partial H_2}{\partial T_{11}} < 0$. That means that the two types of trustors have conflicting interests. Using that trustors of type 2 cannot force trustors of type 1 to give up contacts in their own group for contacts with trustors of type 2, we have the equilibrium as given in the theorem. □

Proof of Theorem 7. We prove assertion i) of the theorem mainly with arguments from Theorem 6. First, we split the trustors in two groups, namely trustors of type 1 and all the others. We calculate for this division the matrix $\boldsymbol{T}$, which has all the properties needed to apply Theorem 6. That means that for optimalization we have to maximize the first column, which can be done by taking all the original elements (before the division of the trustors in two groups) separately optimal. Second we keep the first column fixed and do the same by splitting the whole population of trustors in two groups: the trustors of type 1 and 2 and all the others. Again we have to maximize the first column, which means maximizing the elements of the first two columns in the original matrix. The first column was already fixed. Now the elements of second column have to be taken maximal.

The argument for assertion ii) is similar to that for part i). The difference is that the trustor types with higher numbers depend on trustor types with lower numbers about how much contacts they will have with them. □

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